Studies of the motion and decay of axion walls bounded by strings, and related issues.

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I. INTRODUCTION

The main focus of this paper is the decay into axions of axion walls bounded by axion strings. This process occurs shortly after the QCD phase transition in the early universe if 1) axions exist, 2) the axion field is not homogenized by inflation after the Peccei-Quinn phase transition has occurred, 3) the axion theory has a unique vacuum ($N = 1$; see below). The decay of axion walls bounded by strings produces a cold axion population of density comparable to the cold axion populations produced by vacuum realignment and by axion string decay. The word "cold" indicates that these axions have typical energies which are much smaller than $k_B T$ where T is the ambient temperature.

The axions from vacuum realignment and from axion string decay have, at time $t_1 \sim 2 \cdot 10^{-7}$sec ($T_1 \sim 1$GeV) when the axion mass turn on at the beginning of the QCD phase transition, typical momentum equal to the inverse of the age of the universe then : $\langle p_a \rangle \sim \frac{1}{t_1} \sim \frac{1}{10^{-6}\text{sec}} \sim 10^{-9}$ eV. Here and henceforth $\hbar = c = k_B = 1$. We will find below that the axions produced in the decay of axion walls bounded by string have typical energy $\langle E_a \rangle \sim$ few $\times m_a$ where m_a is the axion mass. Astrophysical and cosmological considerations require the axion mass to be in the range : $10^{-3}\text{eV} \gtrsim m_a \gtrsim 10^{-6}$ eV. The upper limit is derived from the fact that the supernova SN1987a core cooling rate was not shortened by axion emission. The lower limit is derived from the requirement that the energy density in cold axions do not overclose the universe.

In the allowed mass window, the primordial velocity dispersion of axions from wall decay is thus at least a factor 10^3 larger than the velocity dispersion of axions from vacuum realignment and from string decay. This has interesting implications for the evolution of axion mini-clusters. It is also interesting to speculate that the primordial velocity dispersion of cold axions may some day be measured in a cavity-type axion dark matter detector. If a signal is found in such detector, the energy spectrum of dark matter axions can be measured with great resolution. It has been pointed out that the late infall of dark matter axions onto our galaxy produces peaks in the energy spectrum because late infall produces discrete flows, each one with a characteristic velocity vector on Earth. These peaks are broadened by the primordial velocity dispersion. Since axions from vacuum realignment and string decay have $\langle p_a \rangle \sim 10^{-9}$ eV at temperature $T_1 \sim 1$ GeV, their velocity dispersion today is of order $\beta_{a,I} \sim \frac{10^{-9}\text{eV}}{m_a}\frac{T_0}{T_1} \sim 2 \cdot 10^{-17}(\frac{10^{-5}\text{eV}}{m_a})$ where $T_0 \simeq 3$ K is the present temperature of the cosmic microwave background. $\beta_{a,I}$ is the velocity dispersion in intergalactic space. Infall onto our galaxy increases the primordial velocity dispersion by a factor of order 10 because infall compresses the axions in space and hence (by Liouville's theorem) disperses them in velocity space. Thus we arrive at $\beta^E_{a,I} \sim 2 \cdot 10^{-16}(\frac{10^{-5}\text{eV}}{m_a})$ for an estimate of the primordial velocity dispersion on Earth of axions from vacuum realignment and string decay. The corresponding dispersion in energy is $\delta E_{a,I} = \delta(\frac{1}{2}\beta^2 m_a) = \beta\beta_a m_a \sim 2 \cdot 10^{-24}$ eV. The minimum time required to measure $\delta E_{a,I}$ is $(\delta E_{a,I})^{-1} \sim 10$ years, assuming we have ideal measurements and perfectly understand all sources of jitter in the signal. This is quite discouraging. However, the second population of cold axion - those from wall decay - has a larger velocity dispersion by the factor $\frac{\beta_{a,II}}{\beta_{a,I}} \sim$ few $\cdot \frac{m_a}{10^{-9}\text{eV}}$. Hence for the second population, $\delta E^{-1}_{a,II} \sim 10^4$ sec$(\frac{10^{-5}\text{eV}}{m_a})$ which is much less difficult to measure. Thus by measuring the primordial velocity dispersion of axions from wall decay a cavity detector of dark matter axion may some day "look" all the way back to events that took place during the QCD phase transition.

The axion was postulated approximately twenty years ago. Its existence would explain why the strong interactions conserve the discrete symmetries, P and CP in spite of the fact that the Standard Model of the particle interactions as a whole violates those symmetries. The axion in the Nambu-Goldstone boson associated with the spontaneous breaking of the $U_{PQ}(1)$ quasi-symmetry which Peccei and Quinn postulated. Its zero temperature mass is given by:

$$m_a \simeq 6 \cdot 10^{-6}\text{eV} \cdot N \cdot \left(\frac{10^{12}\text{GeV}}{v}\right) \qquad (1)$$

where v is the magnitude of the vacuum expectation value that breaks $U_{PQ}(1)$ and N is a positive integer which describes the color anomaly of $U_{PQ}(1)$. N is also the number of degenerate vacua of the axion model. The axion owes its mass to non-perturbative QCD effects. At temperatures high compared to 1 GeV, these effects are suppressed and the axion mass is negligible. The axion mass effectively turns on when it equals the inverse of the age t_1 of the universe. This happens at a temperature $T \simeq 1$ GeV. Thereafter it increases till it reaches the value given in Eq. (1) which is presumably valid at temperatures below 100 MeV or so.

During the QCD phase transition, each string becomes the boundary of an axion domain wall. The walls bounded by string are unstable and decay. This decay process is the main topic of this paper. We argue that the walls bounded by string decay mainly into barely relativistic axions. We use computer simulations to estimate the typical energy of the radiated axions; our result is : $E_a \sim 7m_a$. We already mentioned that the high velocity dispersion of axions from the decay of walls bounded by string has interesting implications for the evolution of axion miniclusters and for the direct detection of axion dark matter.

To the best of our knowledge, the fact that axions walls bounded by string are likely to decay predominantly into barely relativistic axions was first pointed by two of us in ref. [1]. D. Lyth worked out some of the cosmological implications in ref. [2]. M. Nagasawa and M. Kasawaki performed the first computer simulations of the decay of walls bounded by string and found $E_a \sim 3m_a$ [3]. That their result is smaller than ours is likely due to the fact that their lattice was much smaller than what we used. It should be emphasized that the lattice size which are amenable to present day computers are at any rate puny compound to what one would ideally wish. Indeed the size of the axion string core is of order $\frac{1}{\sqrt{\lambda}v}$ where λ is a coupling constant whereas the axion domain wall thickness is of order m_a^{-1}. The lattice constant must be smaller than $\frac{1}{\sqrt{\lambda}v}$ for the lattice to resolve the string core. The lattice size must be smaller than m_a^{-1} to contain at least one wall. Hence the lattice size must be of order $10\frac{v}{m_a} \times 10\frac{v}{m_a}$ or larger. Present day computers allow lattice size of order 4000×4000, i.e. $\frac{v}{m_a} \sim 100$. On the other hand, in axion model of interest $\frac{v}{m_a} \sim \frac{10^{12}\mathrm{GeV}}{10^{-5}\mathrm{eV}} = 10^{26}$. The computer simulations inform us about the situation of interest only insofar it may be assumed that the physics does not vary dramatically from the case where $\frac{v}{m_a}$ is large to the case where $\frac{v}{m_a}$ is huge. This assumption is plausible but unproven. To try and remedy this weakness, we will try and study the dependence upon $\frac{v}{m_a}$ of the quantities of interest to us, such as E_a/m_a. We will find that E_a/m_a increase very slowly with $\frac{v}{m_a}$, approximately like $\log\left(\frac{v}{m_a}\right)$. If we assume this trend to continue for arbitrarily large $\frac{v}{m_a}$, then $\frac{E_a}{m_a} \sim 50$ for $m_a \sim 10^{-5}$ eV. This would strengthen our findings that the velocity dispersion of axions from wall decay may be measurable in a cavity detector, and that axions from wall decay bind only very loosely to axion miniclusters and are ripped off by galactic tidal forces when the minicluster falls onto a galaxy.

II. COMPUTER SIMULATIONS OF STRINGS UNZIPPING WALLS

We have carried out an extensive program of 2D numerical simulations of domain walls bounded by strings. The Model Lagrangian in finite difference form is

$$
\begin{aligned}
L = \sum_{\vec{n}} \Bigg\{ & \frac{1}{2}\left[\left(\dot{\phi}_1(\vec{n},t)\right)^2 + \left(\dot{\phi}_2(\vec{n},t)\right)^2\right] \\
& - \sum_j \frac{1}{2}\left[\left(\phi_1(\vec{n}+\hat{j},t) - \phi_1(\vec{n},t)\right)^2\right. \\
& \left. + \left(\phi_2(\vec{n}+\hat{j},t) - \phi_2(\vec{n},t)\right)^2\right] \\
& - \frac{1}{4}\lambda\left[(\phi_1(\vec{n},t))^2 + (\phi_2(\vec{n},t))^2 - 1\right]^2 \\
& + \eta\left(\phi_1(\vec{n},t) - 1\right)\Bigg\}
\end{aligned}
\tag{2}
$$

where $\vec{n}$ labels the sites and the sites $\vec{n}+\hat{j}$ are nearest neighbors of $\vec{n}$. In these units ($v \equiv 1$), the wall thickness is $1/m_a = 1/\sqrt{\eta}$ and the core size is $\delta = 1/\sqrt{\lambda}$. Large two-dimensional grids ($\sim 4000 \times 4000$) were initialized with a straight domain wall initially at rest or with angular momentum. The static domain wall solution was obtained by overrelaxation with the Sine-Gordon ansatz $\phi_1 + i\phi_2 = \exp(i\tan^{-1}\exp(m_a y))$ for the strip of width D between the two strings and the true vacuum ($\phi_1 = 1, \phi_2 = 0$) outside. The string and antistring core sites were approximated by $\phi_1 + i\phi_2 = -\tanh(.583r/\delta)\exp(\mp i\theta)$ and held fixed during relaxation. Stable domain walls were obtained for $1/(\delta m_a) \gtrsim 3$. Figure 1 shows a relaxed wall. Arrows in figures are indicating vector (ϕ_1, ϕ_2) and contour line shows the height of axion potential.

A first-order in time and second order in space algorithm was used for evolution with a time step $dt = 0.2$. The boundary conditions were periodic throughout and the total energy was conserved to better than 1%. If the angular momentum was nonzero, then the time derivative $\dot{\phi}$ was obtained by a finite difference over a small time step $dt \sim 0.1$.

The evolution of the domain wall was studied for various values of $1/(\delta m_a)$ and wall length D. The string core attached to the wall is rapidly accelerated by the wall tension and the potential and gradient energies are converted into kinetic energy of the core. An important feature is the increased contraction of the core with speed. For reduced core sizes $\delta/\gamma \lesssim 5$, we noticed a lattice effect, accompanied by 'lattice scraping' and emission of spurious high frequency radiation. This artificial friction eventually balances the wall tension and leads to a terminal velocity. In our simulations we always ensured being in the continuum limit.

We performed a spectrum analysis of the axion radiation produced by collapsing walls using standard Fourier techniques. The two-dimensional Fourier transform is defined by

$$
\tilde{f}(\vec{p}) = \frac{1}{\sqrt{L_x L_y}} \sum_{\vec{n}} \exp\left[2i\pi\left(\frac{p_x n_x}{L_x} + \frac{p_y n_y}{L_y}\right)\right] f(\vec{n}) \tag{3}
$$

for $p_i = 1, \cdots, L_i$ where $(i = x, y)$ with the dispersion relationship $\vec{p}$ is

$$\omega_p = \sqrt{2\left(2 - \cos\frac{2\pi p_x}{L_x} - \cos\frac{2\pi p_x}{L_x}\right) + m_a^2}. \qquad (4)$$

III. CONCLUSIONS

Results from simulations of axion walls bounded by axion strings show that axions from wall decay are barely relativistic and might dominate the axion population after complete decay of walls. Existence of relativistic axion population may change the distribution of axion minicluster and make it possible to detect by background axion detectors.

Even though results from our simulation looks quite promising, still there are a few factors which give significant uncertainties in our estimation of the velocity dispersion.

Between of the prefered value $(m_a\delta)^{-1} \sim 10^{25}$ and the model we used $(m_a\delta)^{-1} \sim 10$, there is a huge gap which cannot be filled by computer simulation. For $\lambda v_a^2/m_a^2 > 100$ simulations show that $\langle E_a\rangle/m_a$ becomes bigger. But the change is at most logarithmic in m_a. If we assume this trend to continue, we estimate $\langle E_a\rangle \sim 50 m_a$ for $m_a = 10^{-5}$ eV.

Another factor which affect the average energy of axions is the time dependence of axion mass around the critical time t_1. We used time independent mass m_a for entire simulation. In real case, however, axion mass is slowly turned on when the QCD phase transition occurs. If we use time dependent mass $m_a(t)$ in the simulation, $\langle E_a\rangle/m_a$ becomes significantly smaller depends on the initial size of the domain wall and the time dependence of axion mass.

These uncertainties make our prediction of velocity dispersion very weak. But we argue that there is a very good chance of detecting a signal from the decay of axion domain wall bounded by strings.

The average energy of axion from domain wall decay is estimated $\langle E_a\rangle \sim N_{\rm DM} m_a$, where $N_{\rm DM}$ should be in the range of factor one or two from order 1. The minimum time required to detect this velocity dispersion today is

$$\delta E_a^{-1} \sim N_{\rm DM}^{-1} \times 10^4 \sec\left(\frac{10^{-5}\rm eV}{\rm m_a}\right),$$

which is very small compared to the time required to detect the velocity dispersion from vacuum misalignment and string decay.

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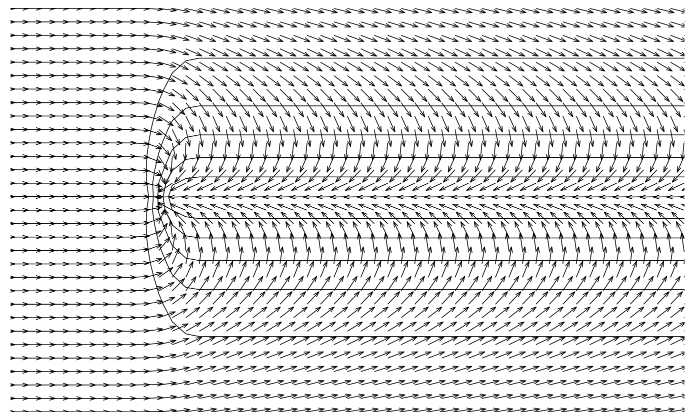

FIG. 1.